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SIMPLE BEAMFORMING TECHNIQUES FOR FULLY ADAPTIVE ARRAYS

by

Eric K.L. Hung

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Research and Development Branch under Project No. 33C69.

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(Radar and Communications Technology Branch)

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ABSTRACT

Presented here are simple techniques to convert beamforming problems for fully adaptive arrays to linear constraint weight vector calculation problems which do not use sample covariance matrices. The techniques to set up the equations for the weight vector are developed with a K-element array. They are then applied to adaptive sidelobe cancellation and the simultaneous nulling of the monopulse sum and difference beam patterns.

The new weight vector calculation method has many desirable properties. It is easy to impose mainbeam constraints on the weight vector. Like the sample matrix inversion (SMI) method, it has rapid convergence characteristics. As well, the equations can be implemented in special-purpose hardware devices such as systolic arrays designed to increase throughput rates in solving linear equations. Finally, the requirements on computer wordlength and arithmetic precision are lower than those of the SMI method.

I. INTRODUCTION

This paper describes simple techniques to convert beamforming problems for fully adaptive arrays to linear constraint weight vector calculation problems which do not use sample covariance matrices. The resultant weight vector calculation has several desirable features. Firstly, it is easy to impose mainbeam constraints on the weight vector. Secondly, the convergence rate is essentially the same as that of the sample matrix inversion (SMI) method [1,2], which has the fastest convergence in all known beamforming methods. Thirdly, like the SMI method, the weight vector equations can be solved with special-purpose hardware devices such as systolic arrays designed to solve linear equations quickly. Finally, the weight vector is calculated directly from the array snapshots and, consequently, the requirements on computer wordlength and arithmetic precision are lower than those of the SMI method, which uses the sums of the products of array snapshots.

The development of the conversion techniques is necessarily heuristic because of various considerations such as mainbeam constraint, convergence rate, computer wordlength, etc.. Hence, the paper has been written essentially in tutorial form with very little mathematics but many examples and references. It is hoped that this presentation will provide a good understanding of the problems to be solved and the manner in which they are handled. For reasons of simplicity, it is assumed that the analogue output of each array element is demodulated through the use of a narrowband bandpass filter and a common local oscillator and then quadra-

ture sampled to produce the complex digital values for weight vector calculation. Detailed descriptions of this data sampling can be found in many references, e.g. [3] and [4].

Beamforming with a fully adaptive array is presented in the next section. Here, the techniques to set up jamming suppression and mainlobe distortion constraint equations are described. A technique to impose a gain constraint on the weight vector calculation is also briefly discussed. Section III describes beamforming with a sidelobe cancellation system. Section IV is concerned with simultaneous nulling of the sum and difference beam patterns in monopulse angle estimation. Section V contains remarks and conclusions.

II. BEAMFORMING WITH A FULLY ADAPTIVE ARRAY

Consider a K-element fully adaptive array as shown in Fig. 1. A

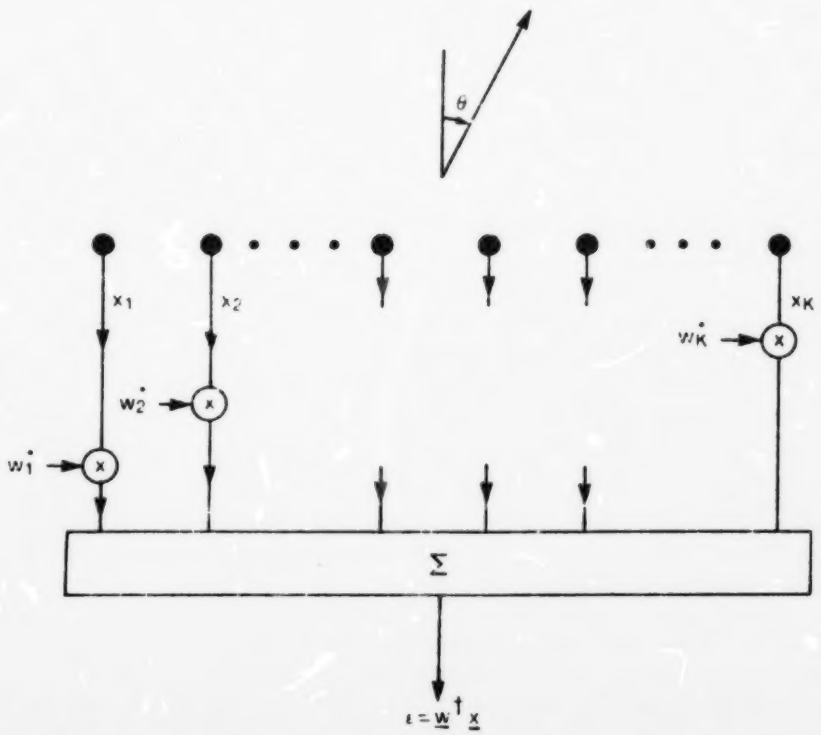

Fig. 1. A K-element fully adaptive array antenn. The outputs of the elements are multiplied by weighting coefficients and summed to produced the array output ε. In the simulations the antenna has K=8 equal-gain isotropic elements spaced at a distance of half a wavelength.

snapshot of the array element output consists of K amplitudes $\{x_1,..,x_k,..,x_k\}$, where x_k is the output of the k-th element. These amplitudes are weighted with coefficients $\{w_1^*,..,w_2^*,..,w_k^*\}$ and summed to give the array output ε. Superscript * denotes complex conjugation. In vector notation,

$$\varepsilon = \underline{w}^\dagger \underline{x} \quad , \tag{1}$$

where

$$\underline{w} = \begin{bmatrix} w_1 \\ w_2 \\ \cdot \\ \cdot \\ w_k \end{bmatrix} \tag{2}$$

is the vector of weighting coefficients and

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_K \end{bmatrix} \tag{3}$$

is a vector of array element output amplitudes. Superscript † denotes the conjugation-transposition operation.

The optimal weight vector for the detection of a signal in noise is proportional to the solution of the equation

$$\underline{R}\,\underline{w} = \underline{v}_s \quad , \tag{4}$$

where $\underline{R}$ is the theoretical covariance matrix of array element output amplitudes and $\underline{v}_s$ the steering vector, or signal direction vector. This equation can be derived with many different performance criteria, including the minimum mean square error [5,6], maximum signal to noise ratio [7-9], and the maximum likelihood ratio [10] criteria. There are many techniques to obtain an estimate of $\underline{w}$. They include the gradient search least mean squares method of Widrow and his coworkers [5,6,11], the maximum signal to noise ratio method developed by Howells [12], Applebaum [7] and Gabriel [13], and the SMI method of Reed, Mallett, and Brennan [1,2]. The SMI method is preferred when rapid convergence is an important requirement. Given a set of M array snapshots $\{\underline{x}_1,\underline{x}_2,\ldots,\underline{x}_M\}$ this method calculates an estimate of the weight vector by solving the equation

$$\hat{\underline{R}}\,\underline{w} = \underline{v}_s \quad . \tag{5}$$

Here, $\hat{\underline{R}}$ is an estimate of $\underline{R}$ and is defined in terms of the snapshots as

$$\hat{\underline{R}} = \frac{1}{M} \sum_{m=1}^{M} \underline{x}_m \underline{x}_m^\dagger$$

$$= \frac{1}{M} \underline{X}\underline{X}^\dagger \quad , \tag{6}$$

where $\underline{X}=(\underline{x}_1,\underline{x}_2,\ldots,\underline{x}_M)$ is a KxM matrix with the array snapshots in its columns. Hudson [14] adds a constant α' to the diagonal elements of $\hat{\underline{R}}$ when it is singular or highly ill-conditioned. The resultant equation for $\underline{w}$ is

$$(\hat{\underline{R}} + \alpha' \underline{I}_K)\, \underline{w} = \underline{v}_s \quad , \tag{7}$$

where $\underline{I}_K$ is a KxK identity matrix. The constant α' is equated to an estimate of the white noise power σ^2 at the outputs of the array elements, because this is the smallest eigenvalue of the theoretical covariance matrix $\underline{R}$.

This section describes a technique to calculate the solution of (7) without calculating $\hat{\underline{R}}$. It will be seen from the derivation of this technique that augmenting the diagonal elements of $\hat{\underline{R}}$ is equivalent to introducing a mainlobe distortion constraint on the weight vector. The derivation also indicates that, in a more general constrained beamforming problem, $\underline{R}$ is not necessarily singular or ill-conditioned and α' need not be an estimate of white noise power.

In the remainder of this section, the notions of jamming suppression and mainlobe distortion constraints are introduced. A method to calculate the least squares weight vector solution of the constraint equations is described. The relationship between this solution and that of (7) is discussed. These are followed by examples of beamforming. A summary is given at the end.

II.A Jamming Suppression Constraint

Let $\underline{w}$ be a non-zero vector orthogonal to every snapshot in the set $\{\underline{x}_1,\underline{x}_2,\ldots\underline{x}_M\}$, i.e.

$$\underline{x}_m^\dagger \underline{w} = 0 \quad , \qquad m=1,2,\ldots,M, \tag{8}$$

$$\underline{X}^{\dagger}\underline{w} = \underline{o}_M \quad ,$$

where $\underline{o}_M$ is a null vector with M components. These two equations are identified here as jamming suppression constraints on the weight vector. Any weight vector which satisfies these constraints effectively separates the space of K-dimensional array output vectors into two subspaces - a signal space and an interference space. The interference space is spanned by the array output vectors used to calculate $\underline{w}$. The component of any snapshot in this space cannot contribute to the array output amplitude. This property can be verified by writing an arbitrary array snapshot $\underline{x}$ as a sum of a signal component $\underline{x}_S$ and an interference component $\underline{x}_I$,

$$\underline{x} = \underline{x}_S + \underline{x}_I \quad . \tag{10}$$

The array output is

$$\begin{aligned} \varepsilon &= \underline{w}^{\dagger}\underline{x} \\ &= \underline{w}^{\dagger}\underline{x}_S + \underline{w}^{\dagger}\underline{x}_I \\ &= \underline{w}^{\dagger}\underline{x}_S \quad . \end{aligned} \tag{11}$$

The interference component $\underline{x}_I$ has no contribution to ε, because it is a linear combination of the $\underline{x}_m$'s and each $\underline{x}_m$ satisfies (8).

II.B Mainlobe Distortion Constraint

The mainlobe distortion constraint is a form of mainbeam constraint. It requires the adapted beam pattern to have the same mainlobe as that of the quiescent beam pattern. There would be no mainlobe distortion if $\underline{w}$ were the same as the weight vector for the quiescent beam pattern. In this study the quiescent weight vector is equated to the steering vector $\underline{v}_s$, $\underline{v}_s = \underline{v}(\theta_s)$, to give

$$\underline{w} = \underline{v}_s \quad . \tag{12}$$

Angle θ_s is the signal direction. There are K constraints in this vector equation, one for each component of $\underline{w}$.

II.C Equation For Weight Vector

The constraint equations (9) and (12) can be combined together to give

$$\begin{bmatrix} \underline{X}^{\dagger} \\ \sqrt{\alpha}\underline{I}_K \end{bmatrix} \underline{w} = \begin{bmatrix} \underline{o}_M \\ \sqrt{\alpha}\underline{v}_s \end{bmatrix} \quad . \tag{13}$$

The adapted weight vector is identified as the least squares solution of this equation. The constant α is a parameter assigned a positive value to control mainlobe distortion. The reasons for restricting it to a positive value will be given later.

It is now shown that the solution of (13) is the solution of (7) multiplied by a scale factor. Let both sides of (13) be pre-multiplied by matrix $(\underline{X}, \sqrt{\alpha}\underline{I}_K)$ and then divided by M. The result is

$$(\hat{\underline{R}} + \alpha'\underline{I}_K)\,\underline{w} = \alpha'\underline{v}_s \quad , \tag{14}$$

with $\alpha'=\alpha/M$. The solution of this equation, hence that of (13), is therefore the solution of (7) multiplied by α'. This scaling of $\underline{w}$ has no effect on the beam shape or output signal to noise ratio.

The above equation indicates that the ratio $\alpha'= \alpha/M$ is a more useful mainlobe distortion parameter than α, because the diagonal elements of $\hat{\underline{R}}$ are increased by this amount. When α' is small compared with the smallest eigenvalue of $\hat{\underline{R}}$, the solution of (14) is approximately proportional to that of (5). The mainlobe distortion constraint is effectively ignored. On the other hand, a value of α' very large compared with the sum of all the eigenvalues of $\hat{\underline{R}}$ reduces (14) to (12); the jamming suppression constraint is effectively ignored.

The reasons for restricting α to a positive value are now given. Firstly, α cannot be zero, otherwise the solutions of both (13) and (14) would be K-dimensional null vectors. Secondly, (14) indicates that the mainlobe distortion constraint plays the same role as white noise. White noise reduces mainlobe distortion by suppressing the correlation of the jamming signals at the array elements. Because white noise power is positive, it is inferred that α should be positive. Consider, finally, what may happen if negative values are allowed. When $-\alpha'$ is close to an eigenvalue of $\hat{\underline{R}}$, for example, $\underline{w}$ becomes essentially an eigenvector of $\hat{\underline{R}}$. The corresponding array gain pattern exhibits peaks in the jammer directions. Consequently, one ends up with jamming exhancement instead of jamming suppression. This property can be exploited in the study of the eigen-spectra of $\hat{\underline{R}}$ and the determination of the jammer directions. However, this study is beyond the scope of this paper.

II.D Simulations

The antenna in the simulations is shown in Fig. 1. It is a linear array with K=8 equal-gain isotropic elements spaced at a distance of half a wavelength. The broadside direction is identified as the signal direction, i.e. θ_s=0 degrees. There are three jammers with bearings -40, 20, and 65 degrees and jammer to white noise ratios 40, 30, and 20 dB, respectively. Equation (13) is solved for $\underline{w}$ by the QR decomposition method. The beam pattern, or dependence of array gain on direction, is calculated as

$$G(\theta) = \frac{|\underline{w}^\dagger \underline{v}(\theta)|^2}{|\underline{w}|^2} \quad , \tag{15}$$

with

$$\underline{v}(\theta) = \begin{bmatrix} 1 \\ \exp[j\pi\sin(\theta)] \\ \vdots \\ \exp[j\pi(K-1)\sin(\theta)] \end{bmatrix} \quad . \tag{16}$$

The definition of $G(\theta)$ leads to unit array gain on white noise, because the expected value of $|\underline{w}^\dagger \underline{x}_N|^2/|\underline{w}|^2$ is σ^2. Here, $\underline{x}_N$ is the noise component in $\underline{x}$, and σ^2 is the average white noise power at the outputs of the array elements. It follows directly from this property that the output powers due to the individual jammers would be less than white noise if the values of $G(\theta)$ are -40, -30, and -20 dB, respectively, in the jammer directions.

The quiescent beam pattern is given in Fig. 2. The arrow above the pattern marks the signal direction. At the bottom are three arrows to mark the jammer directions. The array gains in the jammer directions are -7.8, -4.0, and -9.6 dB, respectively, and are not low enough for jamming suppression.

Presented in Fig. 3 are four adapted beam patterns calculated with (13) and four sets of M=16 array snapshots. The parameter for mainlobe distortion constraint was $\alpha=0.01M\sigma^2$. The patterns indicate that the output powers due to the individual jammers are below white noise. In the 40 dB jammer direction, for example, the array gain is less than -40 dB. However, the patterns are strongly data dependent, have very much distorted mainlobes and rather high sidelobes. The weight vectors used to calculate these patterns can therefore lead to large errors in signal direction estimation. For comparison, adapted beam patterns have also been calculated with (5), the SMI method, and the same four sets of M=16

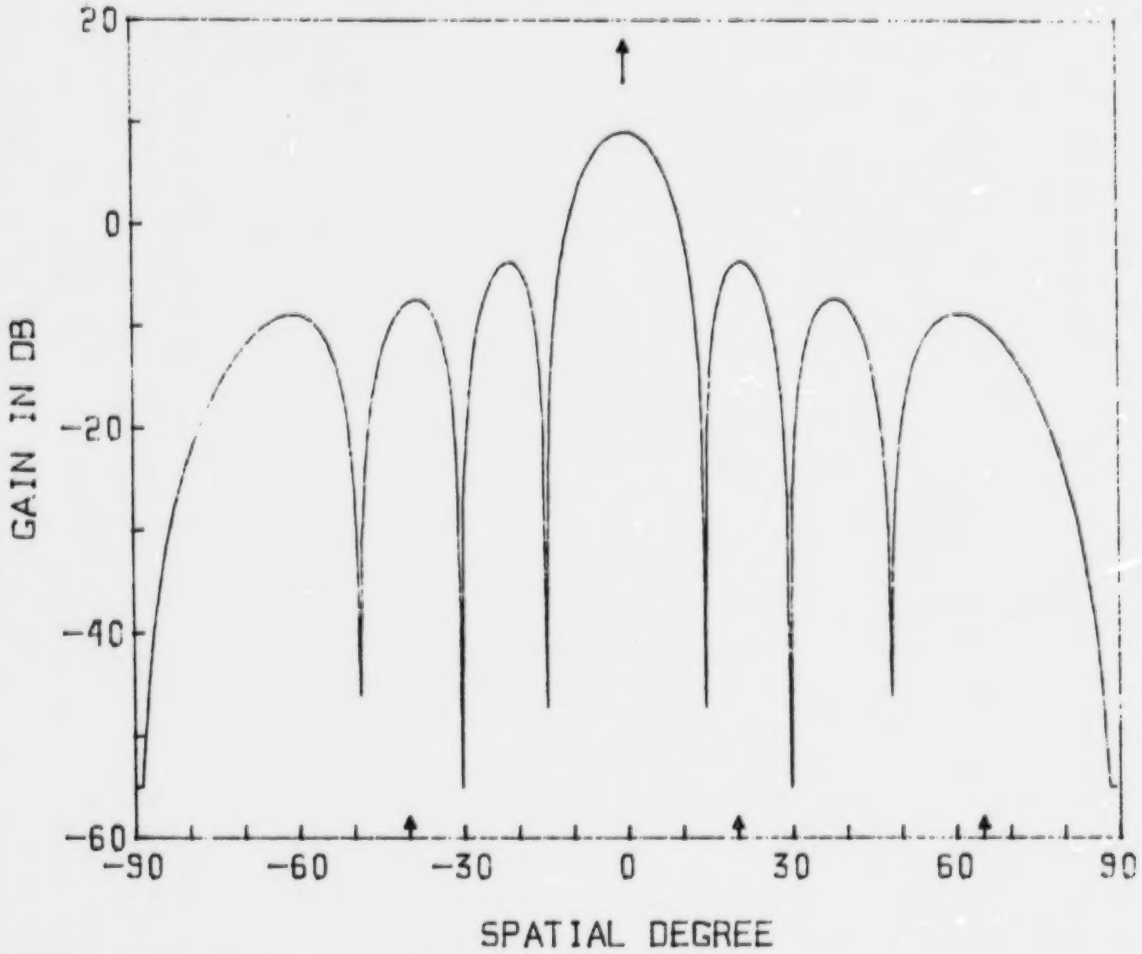

Fig. 2. Quiescent beam pattern of an array antenna with K=8 equal-gain isotropic elements spaced at a distance of half a wavelength. The signal direction is the 0 degree broadside direction. It is marked with an arrow above the pattern. The arrows positions at the bottom are -40, 20 and 65 degrees. They mark the jammer positions in the simulations.

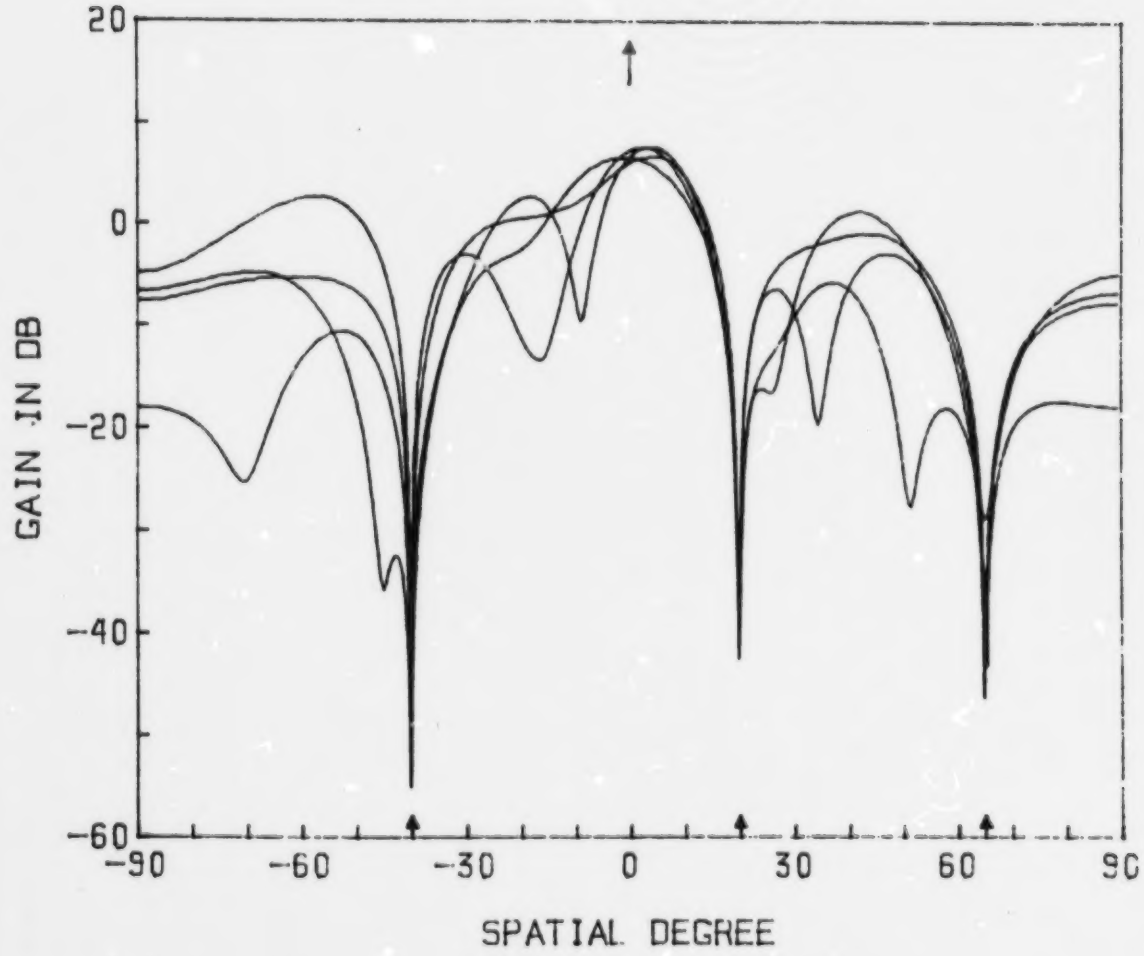

Fig. 3. Four beam patterns constructed with four sets of M=16 array snapshots and $\alpha=0.01M\alpha^2$. These patterns are highly data dependent. Mainlobe distortion is significant.

array snapshots. These patterns are not presented here, because they look identical to those in Fig. 3. Fig. 4 contains four patterns constructed with the same four sets of array snapshots but with $\alpha=100M\sigma^2$. The patterns indicate that the output powers due to the individual jammers are also below white noise. Compared with those in Fig. 3, they are very much less data sensitive and have significantly reduced mainlobe distortions. The average sidelobe level is also very much lower. Note that the sample covariance matrices in these examples are highly ill-conditioned, because the array snapshots are dominated by very strong jammers and the number of these jammers is less than the number of array elements. Ill-conditioned sample covariance matrices are purposely used here, because they produce sharper contrasts between beam patterns calculated with $\alpha=0.01M\sigma^2$ and those calculated with $\alpha=100M\sigma^2$.

In Fig. 5 are beam patterns calculated with (13), M=16 and $\alpha=0.01M\sigma^2$. A 10 dB signal is present in every snapshot. The patterns have nulls in the signal direction, indicating that the signal is treated as a jammer. Furthermore, they are data sensitive and have very high sidelobes. These patterns have been compared with those calculated with the SMI method. There is no visible difference in the patterns. The patterns in Fig. 6 are constructed with the same signal-present array snapshots but with $\alpha=100M\sigma^2$. They are less data sensitive than those in Fig. 5. Mainlobe distortion and sidelobe levels are also significantly reduced. In particular, there is no null in the signal direction.

A few remarks on the mainlobe in Fig. 6 are in order. The shape of this mainlobe is strongly influenced by two opposing constraints. The jamming suppression constraint treats the 10 dB signal as a jammer and tries to suppress it by producing a null in the 0 degrees direction. The mainlobe distortion constraint tries to prevent this. This latter constraint corresponds to an average injected white noise power of 100 ($=\alpha/M$), or 20 dB, at the array elements; evidently, it has overwhelmed the former.

II.E Summary

It has been shown that the weight vector for a fully adaptive array can be calculated with (13). This equation is preferably solved by QR decomposition or other methods which do not involve the computation of the sample covariance matrix $\hat{\underline{R}}$, if computer wordlength and arithmetic precision are important considerations. The parameter α controls mainlobe distortion. It becomes important when α/M is large relative to noise power. In the computer simulations using a uniform linear array, beam patterns which looked the same as those calculated with (5), the SMI method, were obtained when $\alpha=0.01M\sigma^2$ was used. These patterns had very much distorted mainlobes and high sidelobes. Mainlobe distortion and sidelobe levels were significant reduced when α was raised to $100M\sigma^2$.

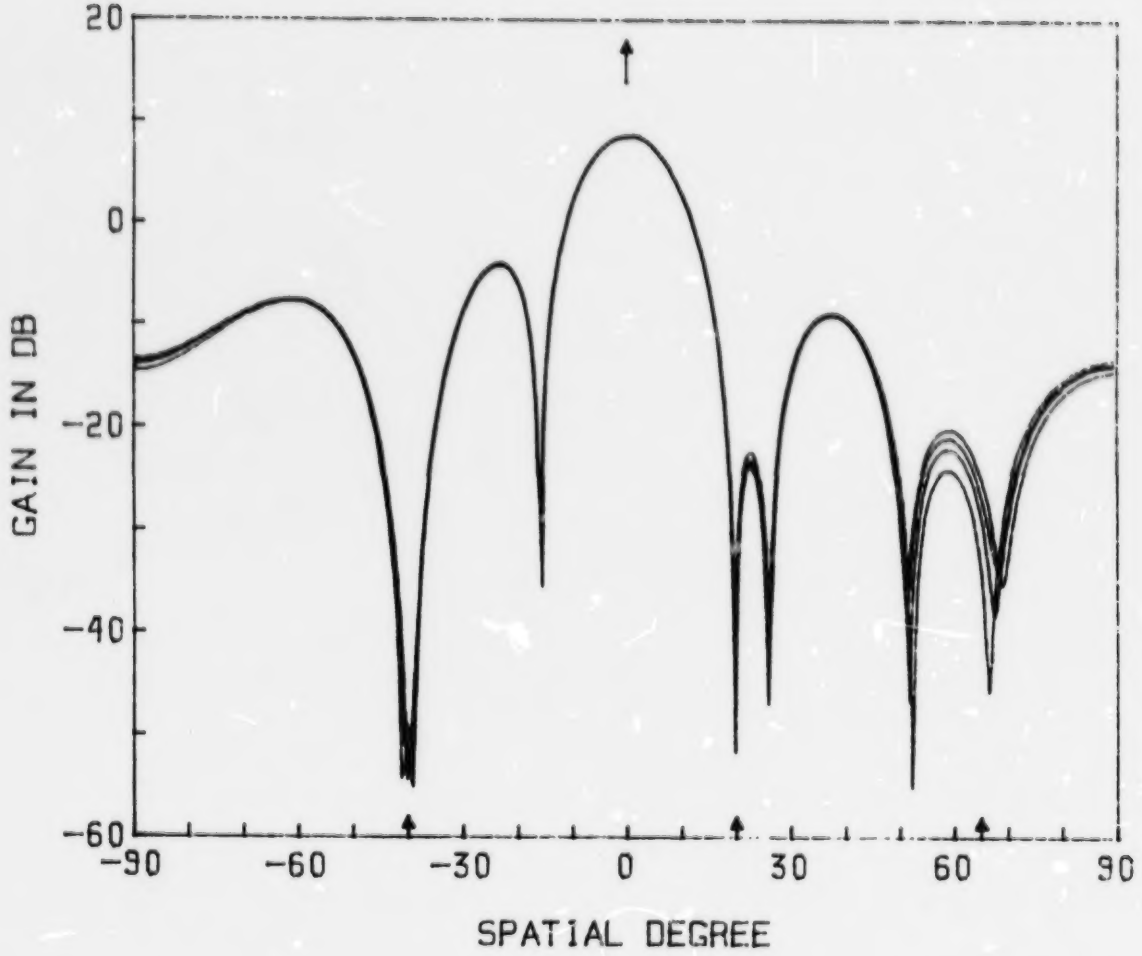

Fig. 4. Four patterns constructed with the same four data sets but with increased to $100M\sigma^2$. These patterns are very much less data sensitive and have significantly reduced mainlobe distortions.

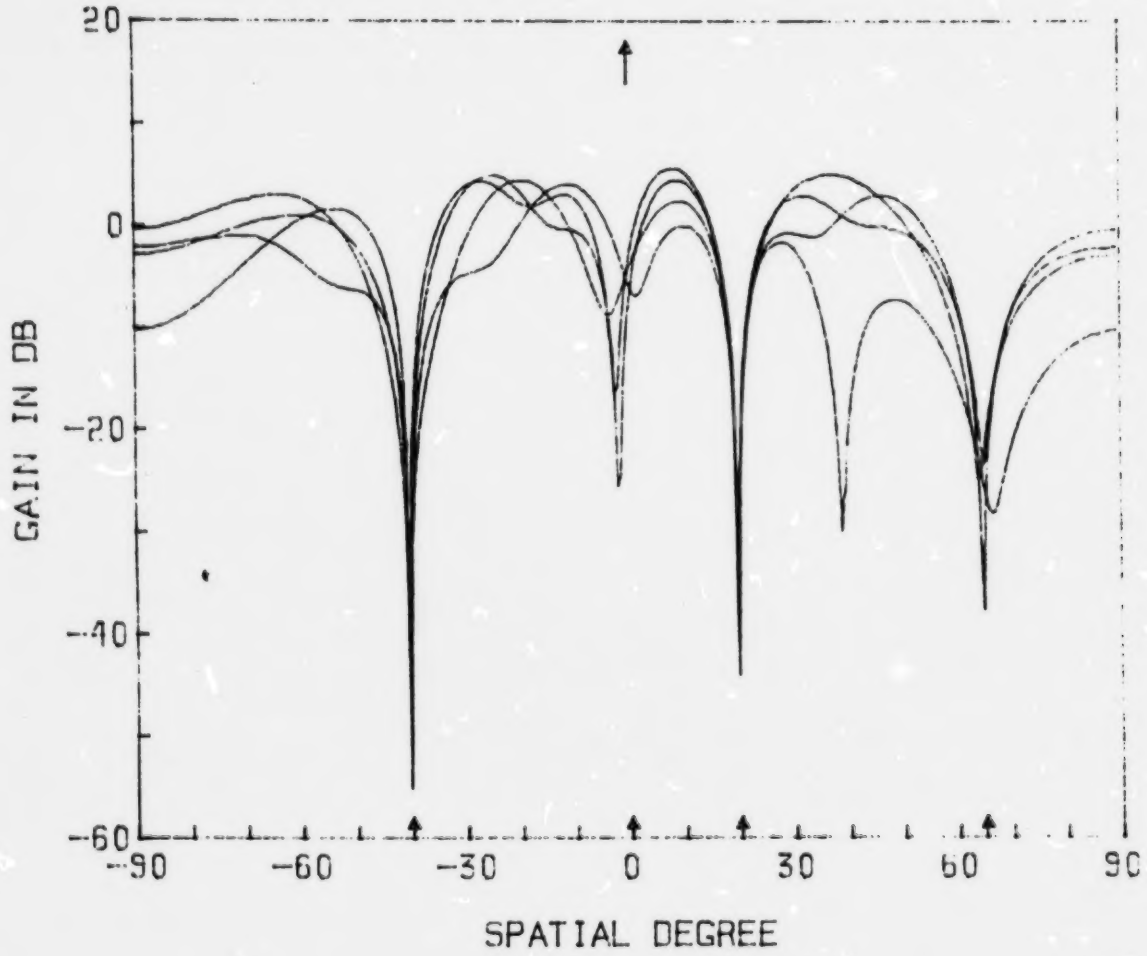

Fig. 5. Four beam patterns constructed with M=16 and $\sigma=0.01M\sigma^2$. A 10 dB signal is present in every snapshot. Nulls are present in the signal direction. These patterns are visibly the same as those calculated with the SMI method.

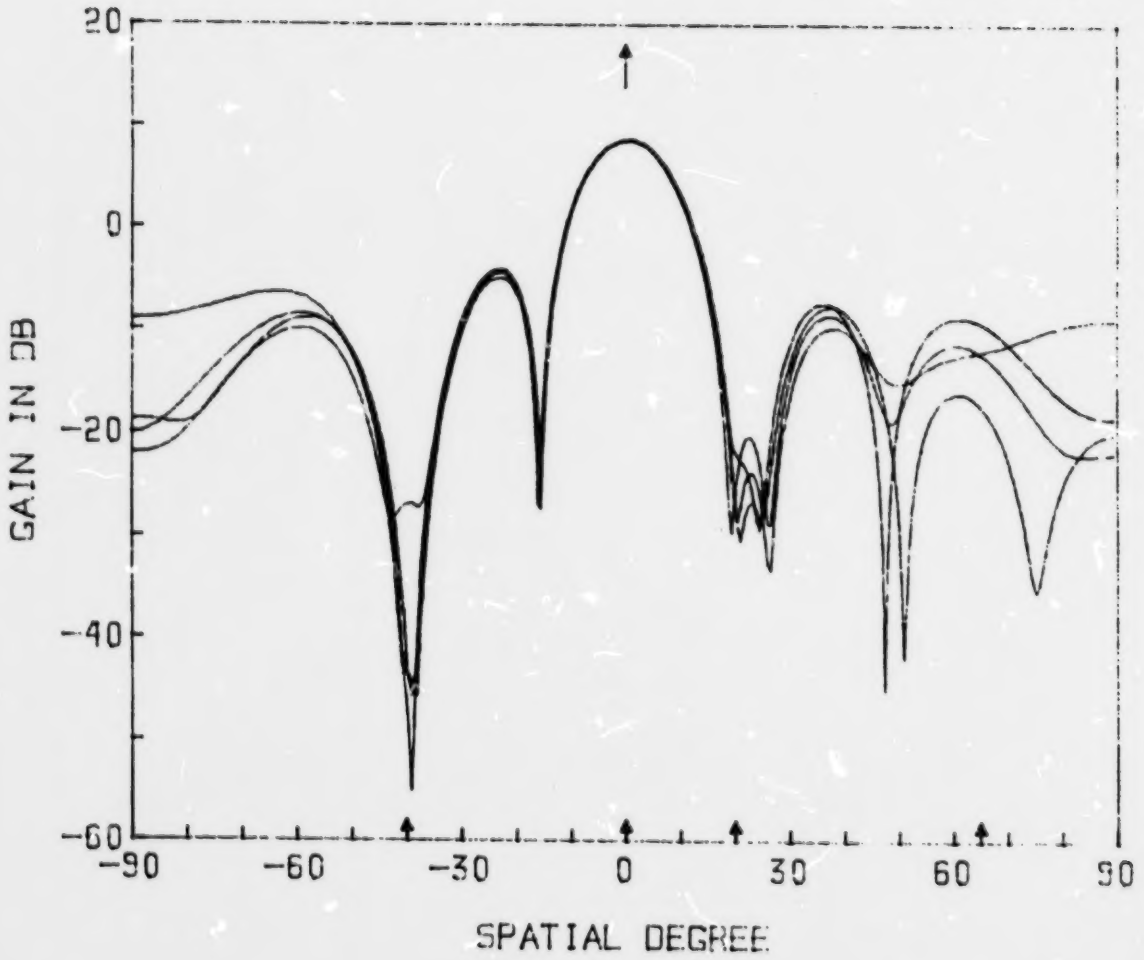

Fig. 6. Four beam patterns constructed with the same four sets of signal-present array snapshots but with increased to $100M\sigma^2$. There is no null in the mainlobe region.

Mainlobe distortion constraint is only a form of mainbeam constraint. It can be replaced by, or combined with, other forms of mainbeam constraints. When it is replaced by the gain constraint

$$\underline{v}_s^\dagger \underline{w} = 1 \quad , \tag{17}$$

for example, the weight vector equation (13) becomes

$$\begin{bmatrix} \underline{X}^\dagger \\ \sqrt{\beta}\underline{v}_s^\dagger \end{bmatrix} \underline{w} = \begin{bmatrix} \underline{0}_M \\ \sqrt{\beta} \end{bmatrix} . \tag{18}$$

Here, β is a parameter such that β/M is very large compared with the sum of all the eigenvalues of $\hat{\underline{R}}$. Similarly, when mainlobe distortion constraint is combined with gain constraint, the weight vector equation becomes

$$\begin{bmatrix} \underline{X}^\dagger \\ \sqrt{\alpha}\underline{I}_K \\ \sqrt{\beta}\underline{v}_s^\dagger \end{bmatrix} \underline{w} = \begin{bmatrix} \underline{0}_m \\ \sqrt{\alpha}\underline{v}_s \\ \sqrt{\beta} \end{bmatrix} . \tag{19}$$

The properties of the adapted beam patterns are highly dependent on the mainbeam constraints, especially when the desired signal is present. However, a detailed study of the various constrained beamforming methods is not the objective of this paper.

III. ADAPTIVE SIDELOBE CANCELLATION

Adaptive sidelobe cancellation is essentially a special case of adaptive beamforming with a fully adaptive array. Discussions of this beamforming method can be found in [7] and [13]. A typical sidelobe cancellation system is shown in Fig. 7. It has a high-gain main antenna accompanied by K isotropic auxiliary antennas. The gains of the auxiliary antennas are approximately equal to the average sidelobe level of the main antenna. The outputs of the auxiliary antennas are weighted, summed, and the sum subtracted from the main antenna output to produce the array output ε. An array snapshot is denoted by a (K+1)-dimensional vector $\tilde{\underline{x}}$, which has an x_0 in the first component and a K-dimensional $\underline{x}$-vector in the remaining components, i.e.

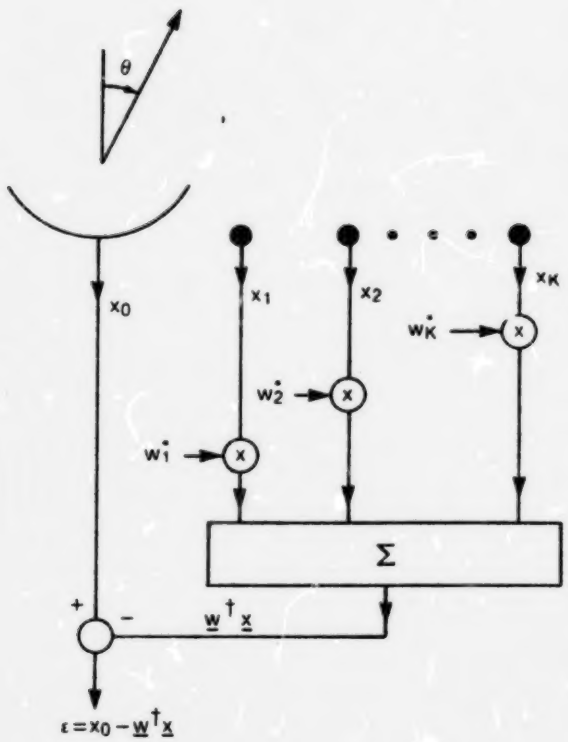

Fig. 7. A sidelobe cancellation system with K axuiliary antennas. The outputs of the auxiliary antennas are weighted, and summed, and the sum subtracted from the main antenna output to produce the array output ε.

$$\tilde{\underline{x}} = \begin{bmatrix} x_0 \\ \underline{x} \end{bmatrix} \tag{20}$$

Here, x_0 is the main antenna output and $\underline{x}$ denotes the outputs of the auxiliary antennas. The array weight vector $\tilde{\underline{w}}$ has unity in the first component and $-\underline{w}$ in the remainder, i.e.

$$\tilde{\underline{w}} = \begin{bmatrix} 1 \\ -\underline{w} \end{bmatrix} \quad . \tag{21}$$

The array output is defined as the Hermitian dot product of $\tilde{\underline{w}}$ and $\tilde{\underline{x}}$,

$$= \tilde{\underline{w}}^{\dagger}\tilde{\underline{x}}$$

$$= x_0 - \underline{w}^{\dagger}\underline{x} \quad . \tag{22}$$

Given a set of M array snapshots $\{\tilde{\underline{x}}_1,\tilde{\underline{x}}_2,\ldots,\tilde{\underline{x}}_M\}$, traditional SMI methods identify the adapted weight vector as the solution of the equation

$$\hat{\underline{R}}\,\underline{w} = \underline{r}_o \tag{23}$$

Here, $\hat{\underline{R}}$ is the sample covariance matrix given by (6),

$$\underline{r}_o = \frac{1}{M}\,\underline{X}\,\underline{b}^* \tag{24}$$

is the sample correlation vector with the k-th component measuring the correlation between output of the main antenna and that of the k-th auxiliary antenna, and $\underline{b}=(x_{01},x_{02},\ldots,x_{0M})^T$ is an M-dimensional vector with x_{0M} denoting the main antenna output in $\tilde{\underline{x}}_m$. When $\hat{\underline{R}}$ is singular or highly ill-conditioned, the corresponding augmented matrix equation for $\underline{w}$ is

$$(\tilde{\underline{R}} + \alpha'\underline{I}_K)\,\underline{w} = \underline{r}_o \quad . \tag{25}$$

The objective here is to develop a technique to calculate $\underline{w}$ without involving the calculation of $\hat{\underline{R}}$.

III.A Jamming Suppression Constraint

Jamming suppression requires $\tilde{\underline{w}}$ to be orthogonal to the array snapshots given. This constraint leads to the set of equations

$$\tilde{\underline{x}}_m^{\dagger}\tilde{\underline{w}} = 0 \quad , \qquad m=1,2,\ldots,M, \tag{26}$$

or, equivalently

$$\underline{X}^{\dagger}\underline{w} = \underline{b}^* \quad . \tag{27}$$

III.B Mainlobe Distortion Constraint

The main antenna beam pattern is identified here as the distortion-free beam pattern of the array antenna. There would be no mainlobe distortion if the auxiliary antennas do not contribute to the array antenna output. Hence the mainlobe distortion constraint is

$$\underline{w} = \underline{\Omega}_K \quad . \tag{28}$$

III.C Equation For Weight Vector

Constraints (27) and (28) combine to give

$$\begin{bmatrix} \underline{X}^{\dagger} \\ \sqrt{\alpha} I_K \end{bmatrix} \underline{w} = \begin{bmatrix} \underline{b}^* \\ \underline{o}_K \end{bmatrix} . \qquad (29)$$

There are more constraints than the number of coefficients in $\underline{w}$. The adapted weight vector is identified as the least squares solution of this equation. The parameter α is a non-negative number used to control mainlobe distortion. It is equated to zero if suppression of mainlobe distortion is not wanted and to a very large value if this suppression is very important.

When both sides of (29) are multiplied on the left by $(\underline{X}, \sqrt{\alpha} I_K)$ the result is (25) with $\alpha' = \alpha/M$. Note that (25) reduces to (23) if $\alpha' = 0$ and effectively to $\underline{w} = \underline{o}_K$ if α' is very large compared with the sum of all the eigenvalues of $\underline{R}$.

III.D Simulations

The sidelobe cancellation system used in the simulation studies is shown in Fig. 8. It is a linear array with (N+K), N=8, equal-gain isotropic antennas spaced at a distance of half a wavelength. The outputs of the first N=8 elements are summed to produce the main antenna output x_o. The (N+k)-th element is identified as the k-th auxiliary antenna. The parameters for the signal and the jammers are the same as those in Section II. The beam pattern is calculated as

$$G(\theta) = \frac{|\tilde{\underline{w}}^{\dagger} \tilde{\underline{v}}(\theta)|^2}{N + |\underline{w}|^2}$$

$$= \frac{|v_o(\theta) - \underline{w}^{\dagger} \underline{v}(\theta)|^2}{N + |\underline{w}|^2} . \qquad (30)$$

Here, $\tilde{\underline{v}}(\theta)$ is the array output due to a unit-amplitude signal in direction qand is given by

$$\tilde{\underline{v}}(\theta) = \begin{bmatrix} v_o(\theta) \\ \underline{v}(\theta) \end{bmatrix} , \qquad (31)$$

with

$$v_o(\theta) = \sum_{n=1}^{N} \exp[j\pi(n-1)\sin\theta] \qquad (32)$$

and

$$v_k(\theta) = \exp[j\pi(N+k-1)\sin\theta] \quad , \quad k=1,2,\ldots,K. \qquad (33)$$

The definition of $G(\theta)$ leads to unit array gain on white noise.

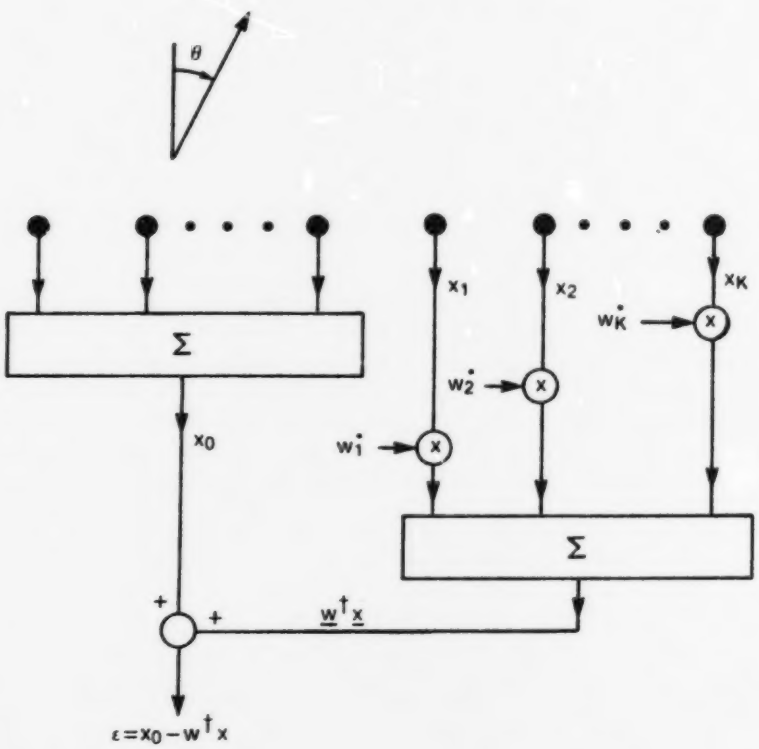

Fig. 8. The sidelobe cancellation system used in the simulation study. It consists of 8+K equal-gain isotropic elements spaced at a distance of half a wavelength. The outputs of the first 8 elements are summed to produce the main antenna output x_o. The (8+k)-th element is identified as the k-th auxiliary antenna.

The distortion-free beam pattern is the 8-element quiescent beam pattern in Fig. 2.

In Fig. 9 are four adapted beam patterns constructed with (29), using four sets of M=16 array snapshots, α=0 for the mainlobe distortion parameter, and K=3 for the number of auxiliary antennas. The patterns have very little mainlobe distortion and very good jamming suppression properties. They are identical to those calculated with (23), the SMI method.

The adapted beam patterns in Fig. 10 are obtained when the the number of auxiliary antennas is increased from K=3 to K=6 to make $\hat{\underline{R}}$ ill-conditioned. These patterns also have good jamming suppression properties. However, they are more data sensitive and have larger mainlobe distortions when compared with those in Fig. 9. The patterns are identical to those obtained with the SMI method. In Fig. 11 are four patterns calculated with the same input data but with the mainlobe distortion parameter increased to $\alpha=100M\sigma^2$. The patterns have reduced mainlobe distortions and are less sensitive to the input data. Their mainlobes are somewhat flattened at the peaks. This flattening is found to depend on the positions and strengths of the jammers.

The effect of α on signal suppression is now studied. In Fig. 12 are patterns constructed with M=16 and α=0. A 10 dB signal is present at 0 degrees in every array snapshot. This signal is treated as a jammer and suppressed, as evidenced by $G(\theta_s)$ being less than -10 dB in the patterns. These patterns are identical to those calculated with the SMI method. Fig. 13 contains patterns constructed with the same signal-present array snapshots but with $\alpha=100M\sigma^2$. The patterns are less data sensitive and the nulls in the signal direction are also not deep enough to suppress the desired signal below white noise.

IV. SIMULTANEOUS NULLING OF MONOPULSE SUM AND DIFFERENCE PATTERNS

In monopulse angle estimation with an array antenna, it is essential that both the sum and the difference beam patterns have nulls in the jammer directions. A straight-forward approach is to use two sets of weighting coefficients, one set to construct the sum amplitude; the other, the difference amplitude. Another method is to construct one set of weighting coefficients and use it for nulling in both the sum and the difference patterns.

A method to calculate a weight vector with a simultaneous nulling property has been described Haupt [15]. It is designed for antennas which do not have provisions for taking array snapshots and, consequently, is restricted to the suppression of jammers in the low sidelobe regions of low sidelobe antennas. An alternative method to calculate a weight vector with a simultaneous nulling property is presented here. This method does not have the above restrictions, because array snapshots are given.

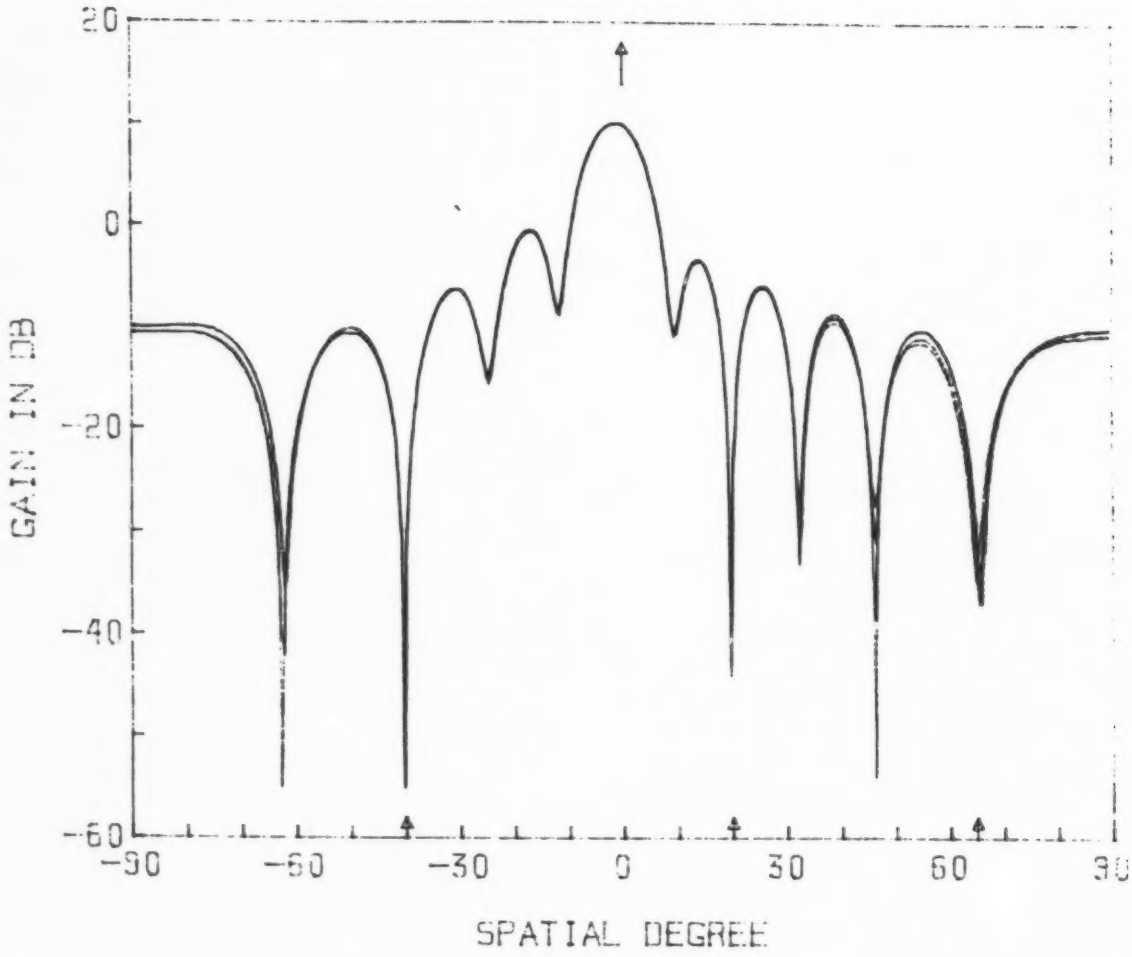

Fig. 9. Four beam patterns constructed with M=16 snapshots, K=3 auxiliary antennas, and α=0 for the mainlobe distortion parameter. These patterns are identical to those calculated with the SMI method.

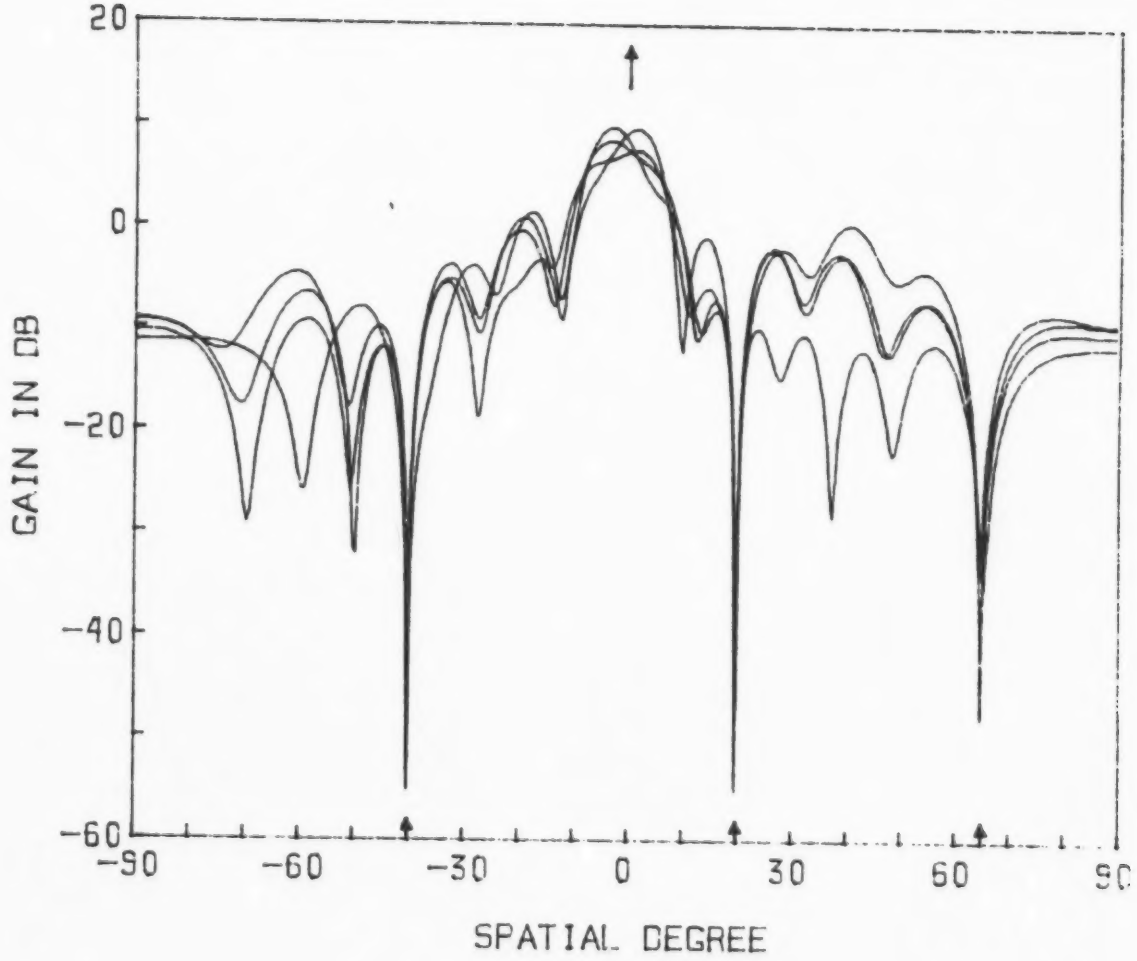

Fig. 10. Four patterns calculated with M=16, α=0, and the number of auxiliary antennas increased from K=3 to K=6. These patterns are identical to those obtained with the SMI method.

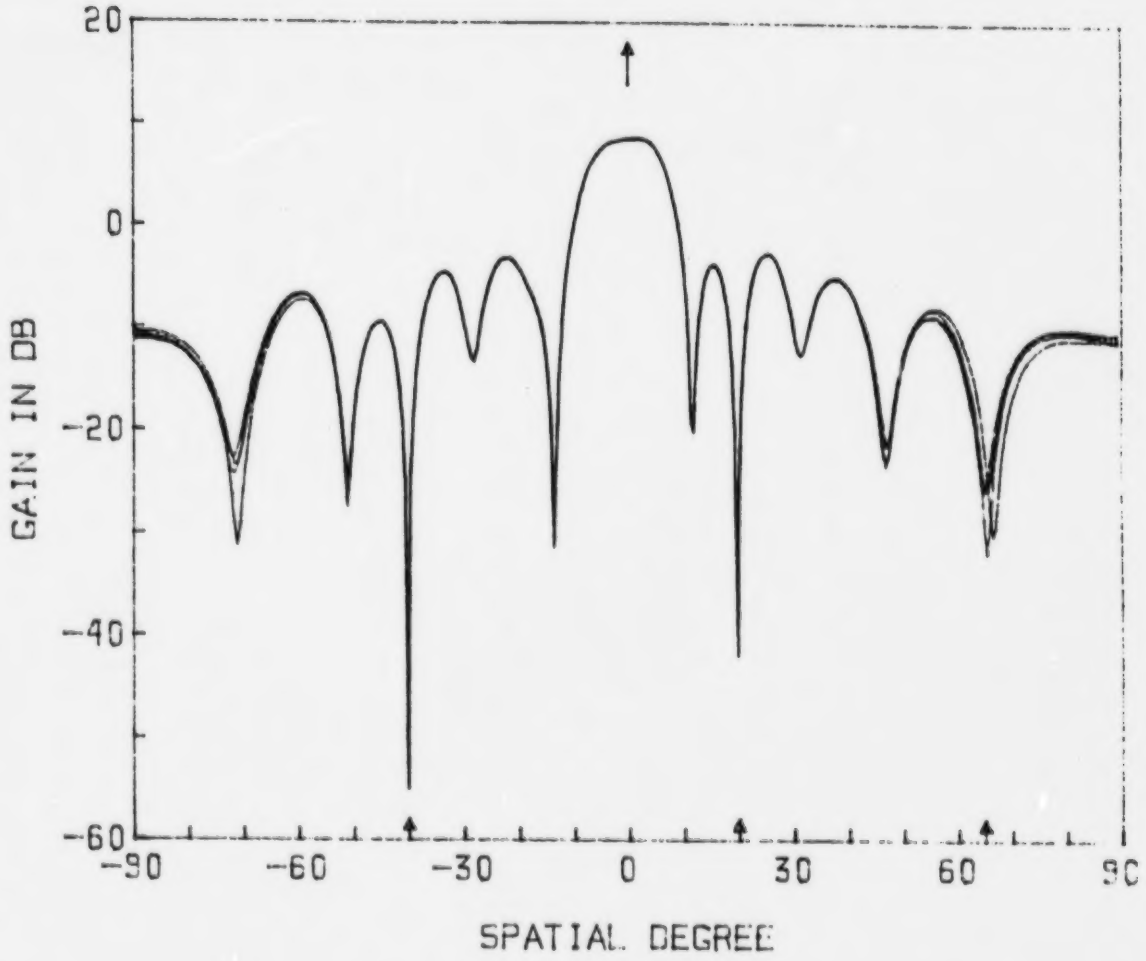

Fig. 11. Four patterns calculated with the M=16 snapshots used in Fig. 10, K=6, and the mainlobe distortion parameter increased to $\alpha=100M\sigma^2$. Compared with those in Fig. 10, the patterns are less data sensitive and have less mainlobe distortions. The flattening on the mainlobes depends on the positions and the strengths of the jammers.

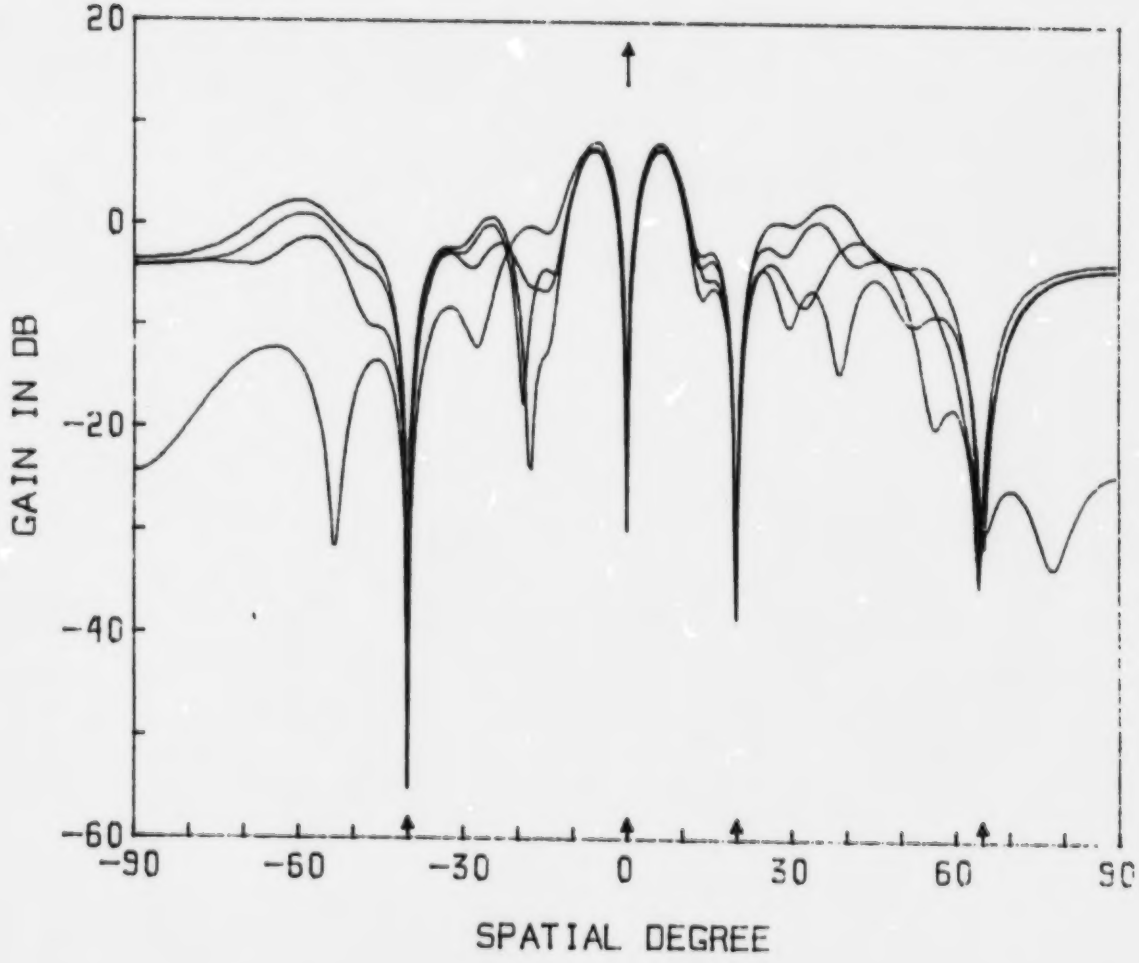

Fig. 12. Four patterns constructed with M=16, K=6, and α=0. A 10 dB signal is present in every snapshot. The signal is treated as a jammer by the beamforming method. These patterns are identical to those obtained with the SMI method.

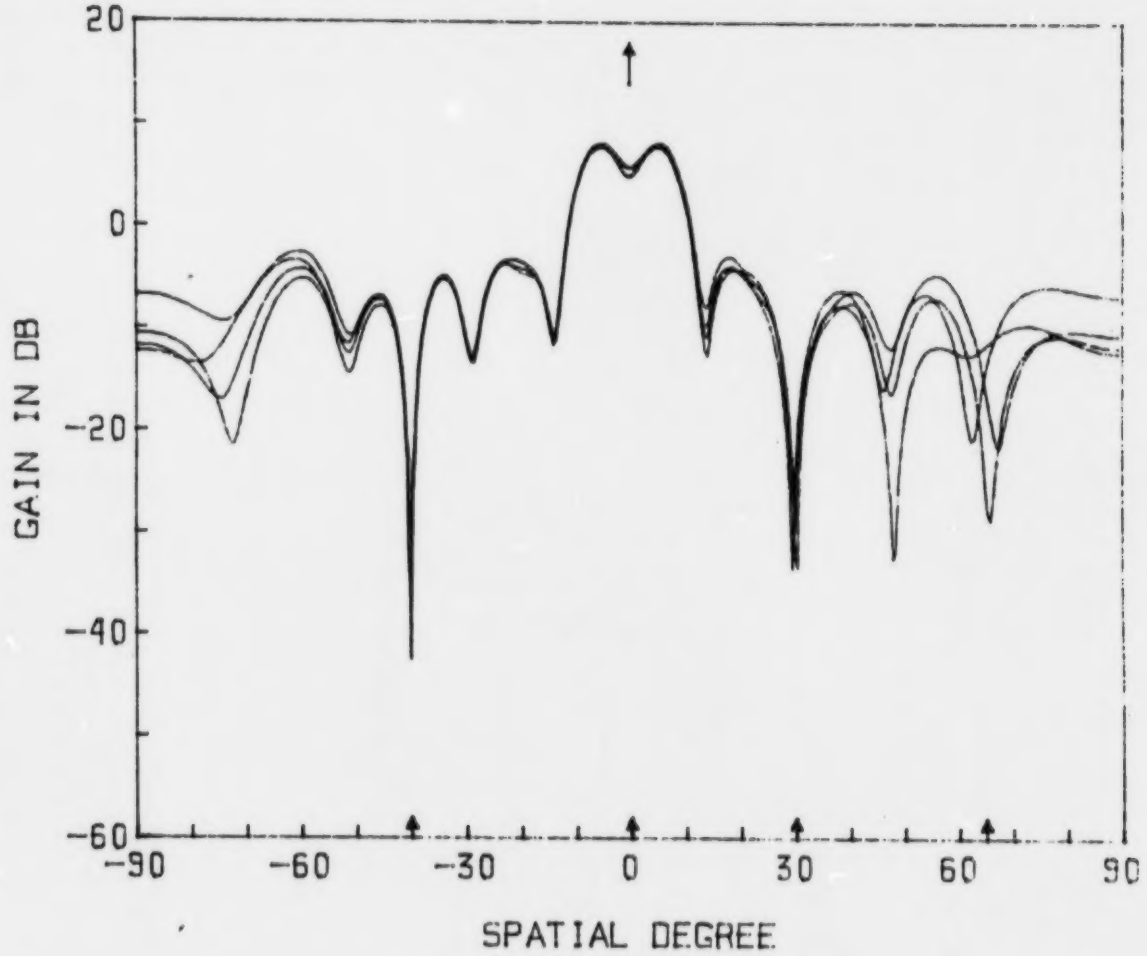

Fig. 13. Patterns constructed with the same M=16 signal-present snapshots, K=6, and increased to $100M\sigma^2$. The patterns are less sensitive to the input data. The null in the mainlobe region of each pattern is no longer deep enough to suppress the desired signal.

An array antenna structured for simultaneous nulling of the sum and difference beam patterns is shown in Fig. 14. It is partitioned into two subarrays, a left array with K' elements and a right array with K" =K-K' elements. The sum and the difference of the subarray output amplitudes are measured and used to determine signal direction. With

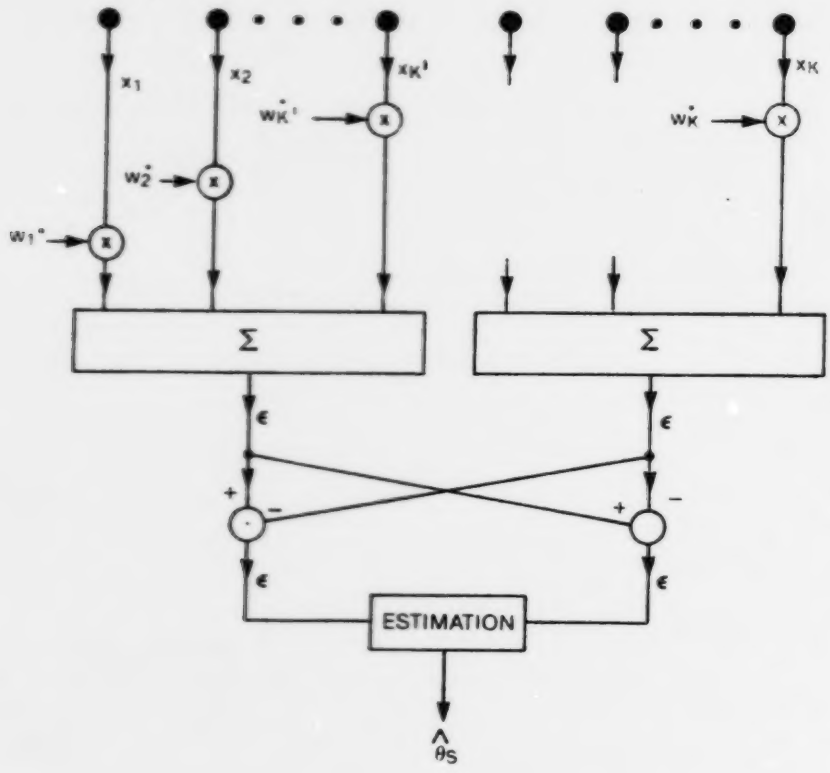

Fig. 14. An array antenna for monopulse angle estimation.

the partitioning of the array antenna, the data matrix $\underline{X}$, the weight vector $\underline{w}_\Sigma$ for the sum amplitude, the weight vector $\underline{w}_\Delta$ for the difference amplitude, and the steering vector $\underline{v}_s$ can be written as

$$\underline{X} = \begin{bmatrix} \underline{X}_L \\ \underline{X}_R \end{bmatrix} , \tag{34}$$

$$\underline{w}_\Sigma = \begin{bmatrix} \underline{w}_L \\ \underline{w}_R \end{bmatrix} , \tag{35}$$

$$\underline{w}_{\Delta} = \begin{bmatrix} \underline{w}_L \\ -\underline{w}_R \end{bmatrix} , \tag{36}$$

and

$$\underline{v}_s = \begin{bmatrix} \underline{v}_{sL} \\ \underline{v}_{sR} \end{bmatrix} . \tag{37}$$

Subscripts L and R signify the quantities associated with the left and the right subarrays, respectively. The size of matrix $\underline{X}_L$ is K'xM and that of $\underline{X}_R$ is K"xM. Vectors $\underline{w}_L$ and $\underline{v}_{sL}$ have K' components and $\underline{w}_R$ and $\underline{v}_{sR}$ have K" components.

IV.A Jamming Suppression Constraints

The jamming suppression constraints are

$$\underline{X}^{\dagger}\underline{w}_{\Sigma} = \underline{o}_M \tag{38}$$

for the sum beam and

$$\underline{X}^{\dagger}\underline{w}_{\Delta} = \underline{o}_M \tag{39}$$

for the difference beam.

IV.B Mainlobe Distortion Constraints

The constraints are the same for the sum and difference beam patterns and are given by

$$\underline{w}_L = \underline{v}_{sL} , \tag{40}$$

and

$$\underline{w}_R = \underline{v}_{sR} . \tag{41}$$

IV.C Equation For Weight Vectors

Equations (34) to (41) lead to

$$\begin{bmatrix} \underline{X}^{\dagger} \\ \underline{X}_{\Delta}^{\dagger} \\ \sqrt{\alpha}\underline{I}_K \end{bmatrix} \begin{vmatrix} \underline{w}_L \\ \underline{w}_R \end{vmatrix} = \begin{bmatrix} \underline{o}_M \\ \underline{o}_M \\ \sqrt{\alpha}\underline{v}_s \end{bmatrix} . \tag{42}$$

Matrix $\underline{X}_\Delta$ is defined as

$$\underline{X}_\Delta = \begin{bmatrix} \underline{X}_L \\ -\underline{X}_R \end{bmatrix} . \qquad (43)$$

Parameter α controls mainlobe distortion. The least squares solution of (42) gives rise to $\underline{w}_L$ and $\underline{w}_R$, hence $\underline{w}_\Sigma$ and $\underline{w}_\Delta$.

IV.D Simulations

The antenna, signal direction, and jammer parameters are the same as those in the simulation study in Section II. The left subarray comprises the elements labelled with k=1 to k=4. The array gain is calculated with (15), using $\underline{w}=\underline{w}_\Sigma$ for the sum pattern and $\underline{w}=\underline{w}_\Delta$ for the difference pattern.

A pair of sum and difference patterns is shown in Fig. 15. It is constructed with one set of M=16 snapshots and $\alpha=100M\sigma^2$. The sum pattern is the solid curve; the difference, the dashed curve. Both patterns have sufficient null depths to suppress the output powers due to the individual jammers below white noise. Note that the ratio of the gain in the sum pattern to the gain in the difference pattern is given by the vertical distance from the dashed curve to the solid curve. This ratio has an absolute maximum close to the signal direction θ_s=0 degrees.

IV.E Remarks

A set of two decoupled equations

$$(\hat{\underline{R}}_L + \alpha' \underline{I}_{K'})\, \underline{w}_L = \alpha' \underline{v}_{sL} \qquad (44)$$

and

$$(\hat{\underline{R}}_R + \alpha' \underline{I}_{K''})\, \underline{w}_R = \alpha' \underline{v}_{sR} \qquad (45)$$

is obtained when both sides of (42) are multiplied on the left by $(\underline{X}, \underline{X}_\Delta, \sqrt{\alpha}\underline{I}_K)$ and then divided by 2M. Here, $\alpha'=\alpha/2M$, and covariance matrices $\hat{\underline{R}}_L$ and $\hat{\underline{R}}_R$ are related to $\underline{X}_L$ and $\underline{X}_R$, respectively, by (6). The equations indicate that the beamforming method treats each subarray as a fully adaptive array, independent of the presence of the other. The maximum number of jamming signals the left and the right subarrays can suppress below white noise are (K'-1) and (K"-1), respectively. Hence, the maximum number of jamming signals a K-element array can suppress is the smaller of (K'-1) and (K"-1), or three jamming signals with a K=8 element array and K'=K"=4.

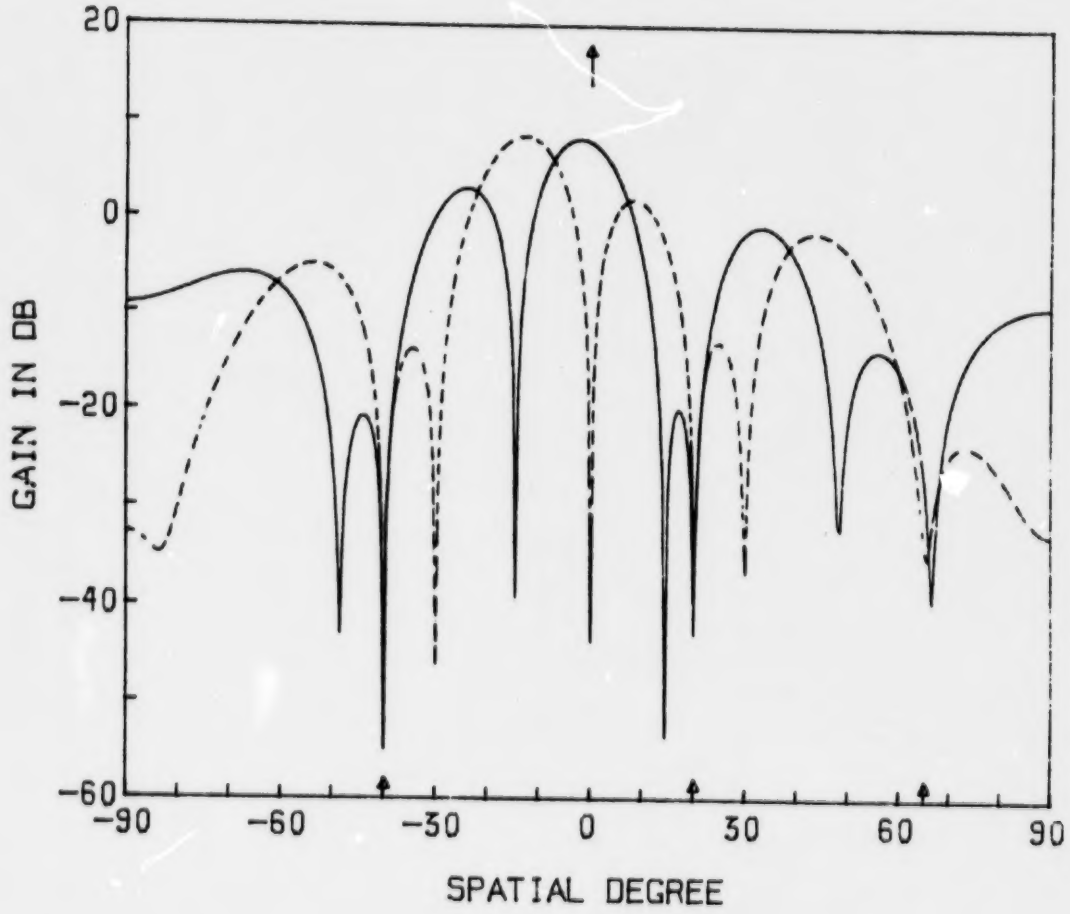

Fig. 15. A pair of sum and difference beam patterns constructed with K=8,M=16, and $\alpha=100M\sigma^2$. The sum pattern is the solid curve; the difference, the dotted curve. The vertical distance from the dotted curve to the solid curve is the ratio of the sum power to the difference power.

Monopulse angle estimation can also be carried out with separate sets of weighting coefficients to calculate the sum and the difference amplitudes. In this case, $\underline{w}_\Sigma$ is the $\underline{w}$ calculated with (13) and $\underline{w}_\Delta$ is the least squares solution of

$$\begin{bmatrix} \underline{X}_\Delta^\dagger \\ \sqrt{\alpha}\underline{I}_K \end{bmatrix} \underline{w} = \begin{bmatrix} \underline{0}_M \\ \sqrt{\alpha}\underline{v}_s \end{bmatrix} . \qquad (46)$$

The maximum number of jamming signals the antenna can suppress is (K-1), or seven jammers with a K=8 array antenna. This is larger than the number of jammers in the case of simultaneous nulling, because $\underline{w}_\Sigma$ and $\underline{w}_\Delta$ are no longer related by (35) and (36).

V. CONCLUSIONS

Techniques to convert beamforming problems for fully adaptive array antennas to linear constraint weight vector calculation problems which do not involve the use of sample covariance matrices have been described. The examples given are beamforming with a K-element array, beamforming with a sidelobe cancellation system, and the simultaneous nulling of the monopulse sum and difference patterns. Three types of constraints have been discussed. The first, jamming suppression constraint, involves finding a weight vector orthogonal to every vector in a given set of array snapshots. The second, mainlobe distortion constraint, steers the main beam of the adapted pattern at the signal direction. It is a form of mainbeam constraint. The third, gain constraint, constrains the array gain to have a specified value in a specified direction. The imposition of this constraint was only briefly mentioned.

The equations for the weight vector have an adjustable parameter to control the level of mainlobe distortion constraint. When this parameter is assigned a very small value, mainlobe distortion constraint is suppressed and the resultant beam patterns look the same as those obtained with the traditional SMI method.

The examples given demonstrated that the new approach has many favourable features. Firstly, it is easy to impose mainbeam constraints on the weight vector. Secondly, the convergence is rapid. Thirdly, the weight vector equations can be solved with special-purpose hardware devices such as systolic arrays designed to solve linear equations quickly. Finally, the requirements on computer wordlength and arithmetic precision are lower than those of the SMI method.

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Presented here are simple techniques to convert beamforming problems for fully adaptive arrays to linear constraint weight vector calculation problems which do not use sample covariance matrices. The techniques to set up the equations for the weight vector are developed with a K-element array. They are then applied to adaptive sidelobe cancellation and the simultaneous nulling of the monopulse sum and difference beam patterns.

The new weight vector calculation method has many desirable properties. It is easy to impose mainbeam constraints on the weight vector. Like the sample matrix inversion (SMI) method, it has rapid convergence characteristics. As well, the equations can be implemented in special-purpose hardware devices such as systolic arraysdesigned to increase throughput rates in solving linear equations. Finally, the requirements on computer wordlength and arithmetic precision are lower than those of the SMI method.

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